Legal knowledge based systems
JURIX 91
Model-based legal reasoning

The Foundation for Legal Knowledge Systems
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C.N.J. De Vey Mestdagh, W. Verwaard, J.H. Hoepman, The logic of reasonable inferences: J.A.P.J. Breuker, R.V. De Mulder, J.C. Hage (eds.), Legal knowledge based systems JURIX 91: Model-based legal reasoning , The Foundation for Legal Knowledge Systems, Lelystad: Koninklijke Vermande, pp. 60-76, 1991 ISBN 90 6040 989 2.

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THE LOGIC OF REASONABLE INFERENCES

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Summary

The body of rules of law, evolving from the expression of conflicting opinions inherent in the legal system, introduces inconsistency as an element of the legal theory which describes the process of drawing legal conclusions. The logic of reasonable inferences proposed in this paper will serve as a model for the justification of legal decisions and should therefore incorporate this meaningful inconsistency. The semantics of the logic will be defined, taking the semantics of predicate calculus as a starting-point. Inconsistency is handled by allowing the derivation of conflicting conclusions, whose justifications differ in the choice between mutually inconsistent rules. These choices and their resulting conclusions induce various so-called contexts, the number of which can be reduced at a subsequent stage by the application of meta-norms. The authors are at the present engaged in the development of a rule-based expert system for the enforcement of Dutch environmental permit law, the inference part of which is specified by the proposed logic. Some implementational aspects of this Inference Engine will be considered in this paper.

I Introduction

The legal system is characterized by conflicting opinions. The legislator can not foresee every factual situation in which a legal norm should apply. Furthermore, he is not able to anticipate all of the changes in social opinions which may lead to dissenting interpretations of legal norms. Finally, the legal system is based on the idea that certain actors play opposing roles in the process of finding justifications for legal conclusions. The actors of the legal system defend different interests and therefore stress the importance of different factual circumstances, support different qualifications of the same facts, use different interpretation rules for open-textured concepts and adhere to different policy rules to fill in the decision space. They even introduce different social opinions as premisses for their justification of certain legal conclusions.

The legislator expresses these aspects of the legal system by using open-textured concepts and by granting decision space. The jurisdictor and the administrator interpret these concepts and use this decision space to adjust the law to particular circumstances and to the interests they represent, thus introducing interpretation rules and policy rules which can be regarded as rules of law and therefore as part of the legal system. In cases of a strong demand caused by changed social opinions, they even make decisions which dissent from the statutory rules of law.

The body of rules of law, evolving from the expression of conflict inherent in the legal system, introduces inconsistency as an element of the legal theory which describes legal reasoning. This meaningful inconsistency must be handled by a system which models legal reasoning. The system should be able to generate **all** conclusions derivable from the rules of law and the factual situation, using different policy rules, and exploiting the decision space. This way the system will be able to aid all actors in the legal system.

Any conclusion will be drawn using only a subset of all rules of law. This subset is called a *justification*, which should be internally consistent (cf. [Nie76], [Cro77]). Since the ultimate choice of a viable conclusion is limited by so-called meta norms, which give preference to certain justifications, the system must also be able to generate all (alternative) justifications for the same conclusion.

Our research is aimed at the development of a rule-based expert system to aid the enforcement of Dutch environmental permit law ([Vey90]). One of the main components of this expert system is an *inference engine*, which executes the rulebases. The inference engine is composed of two independent stages: the *first stage* generates all possible conclusions and their *justifications* using the rules supplied by the rulebase, and the *second stage* will guide the user in the selection of an appropriate conclusion and the related justification (which together are called a *context*) by applying meta norms. This second stage is as yet hypothetical, but the first stage has been implemented.

For the formal specification of the first stage, we require a logical framework modelling legal reasoning. To this end, we propose the logic of reasonable inferences. This logic should provide means to handle inconsistent assumptions, multiple conclusions, and alternative justifications, and should incorporate the possibility to compare these by using meta rules. For more details concerning our research and the expert system developed see [Laa88], [Vey88], [Vey89a], [Vey89b], [Vey89c], [Vey90].

Although there are many sources of contradiction in the legal system, of which the major examples are alternative opinions, exceptions, introduction and changing of rules of law, and changes in the factual situation, only alternative opinions provide a source of meaningful inconsistency. In all the other cases there exists a rather simple principle to resolve the contradiction. Exceptions overrule their general rules, new rules and facts override their predecessors. Resolving the contradictions resulting from alternative opinions demands expertise and in many cases authority combined with knowledge of dominant social opinions. The expertise consists of knowledge of a set of meta rules which are part of the legal system. Some of these rules are coercive but most of them are only tentative of nature.

An example from Dutch environmental law

The legal system provides us with many examples of the described contradictions. In this paper an example from Dutch environmental law will be used as an illustration of the theoretical problem of contradiction (in this section), to outline some properties of the formal description (in section 3) and to explain some features of the implementation (in section 4).

The Dutch Waste Products Law (WPL) obliges industries which "handle waste" to do so following the directions of an environmental license (section 31 WPL). The ambiguous concepts "handle" and "waste" and their combination have caused a vast body of rules of interpretation, which in many cases contradict each other. These contradictions have been the subject of many subsequent legal disputes. This can be elucidated with the example of rubble. Until 1980, according to common legal opinion, rubble was designated as "waste" and any use of rubble was labeled as "handling of waste". In 1981 this interpretation was refined by the "Kroon" (the highest body of administrative appeal). Waste was defined as any product which is no longer used for a specific purpose (KB may 29 1981, BR 1982, p. 69), thus introducing an exception to the general rule. In a specific case this meant that a farmer who used rubble to fill up a ditch, and thus not just dumping the rubble but using it to attain a purpose, did not need a WPL license. Some months later the "Hoge Raad" (the highest body of civil appeal) decided that common parlance should be the criterion for the judgement of the waste property of any product (HR december 22, 1981, NJ 1982, 325). According to this interpretation a WPL license was needed in any case concerning rubble, even if it was used to fill up a ditch. As there are no hierarchy regulations which grant higher authority to the opinions of either of this bodies of appeal, both interpretations were valid within the legal system at the same time. Although, in this specific case, there exists a meta rule stating that a court of law should adhere to its own previous jurisdiction, this meta rule is not coercive but tentative by nature. This provides us with one of the many clear cut examples of alternative legal opinions, which can be used at will in cases coming up in any court of law. This was confirmed by the refusal of the "Kroon" to obey a directive from the minister for the environment to adjust to the jurisdiction of the "Hoge Raad" (UCV 32, december 10, 1984, p. 12/13). The conflict was finally resolved by legislation. Section 31 clause 3 WPL jo "Werkenbesluit Afvalstoffenwet" declares rubble to be waste under any circumstance or use. In this case a coercive meta rule exists preferring legislation to jurisdiction. However the conflict remains for any other material (except rubble) for which no specific definitions are contained in the new legislation. So the interpretation rules of both the "Kroon" and the "Hoge Raad" are still valid except for rubble. This means that the rulings of the "Kroon" and the "Hoge Raad" cannot be removed from the rulebase. The introduced rule of law constitutes an exception to the present rules and calls for revision of some registered cases concerning rubble. Revision of registered cases is also needed if relevant factual circumstances change. In the rubble case, the farmer can dump some material designated waste, for instance wreckage, on top of the filled up ditch. This requalifies the rubble as waste according to common legal opinion (which should be comprised in the rulebase).

Section 2 provides an overview of some relevant topics on logics proposed as models of automated reasoning, and should indicate the position of our theory within that research. Section 3 defines the logic of reasonable inferences, illustrates the behaviour of the logic with some examples and lemmas, and shows how the logic of reasonable inferences extends Poole's framework for default reasoning. Section 4 discusses some implementational aspects of the logic. This paper concludes with a short discussion of further research in section 5.

II State of the art

For some of the sources of contradiction in the legal system there exist formal and technical approaches which resolve the resulting inconsistencies. However, this is not case for alternative opinions which result in meaningful inconsistency and therefore should be formally and technically specified as *reasonable inferences* and their justifications. The contradictions resulting from alternative opinions should not be resolved but have to be explored by legal knowledge based systems. Finally a formal description and a technical implementation of the generation of contexts based on alternative opinions should allow the application of meta rules to guide the jurist in his decision which opinion to prefer.

Checking the rulebase for inconsistencies and removing the sources of inconsistency, or aborting the reasoning process if an inconsistency is encountered is too restrictive for our purposes, since the rules representing alternative opinions and general rules and their exceptions should be part of the rulebase.

Non monotonic reasoning is another approach to handling inconsistency. It is concerned with "the derivation of plausible (but not infallible) conclusions from a knowledge base. [..] Any such conclusion is understood to be tentative; it may have to be retracted after new information has been added to the knowledge base" [Rei87]. The notion of non monotonic reasoning has been worked on within several formal frameworks, in particular Circumscription ([MCar80], [MCar86]), Default Reasoning ([Rei80], [Rei81], [Poo88], [Eth87a], introducing the notion of default rules) and Non-Monotonic Logic (a modal approach is proposed by [MDer80], introducing a modality, denotating the notion of consistence). Belief revision (also called truth maintenance or reason maintenance, [Doy79], [Kle86a], [Kle86b], [Kle86c]) allows a problem solver to make nonmonotonic inferences using constraint satisfaction to determine what data is to be believed. It can be considered as a means to optimize the derivation of all possible solutions.

Much work has been carried out to capture different frameworks in a single framework. Etherington [Eth87b] gives some conditions for the subsumption of Circumscription under Default Reasoning. In [Sho87] a framework is presented which subsumes most of the semantics of non-monotonic logics. The logic of reasonable inferences actually shares some important aspects with Poole's framework for default reasoning [Poo88], but differs in the formal specification of its semantic derivability relation and in its proposed use. This observation will be elaborated upon in section 3.3.

A third approach uses certainty factors to assign and handle the amount of (un)-certainty or belief in assumptions. Inconsistency is avoided by assigning one certainty factor to an assumption instead of allowing that assumption and its negation to occur simultaneously. This approach does not allow inconsistent assumptions and alternative derivations, but merely avoids them altogether.

Constraint-satisfaction in general (allowing for general rules which rule out possible solutions) does allow multiple solutions and contradictory assumptions to exist. We do not consider this a viable approach for the inference process, since the only problem-specific constraint present in the first stage of our inference engine is the constraint that all produced justifications should be internally consistent. Constraint satisfaction might however be applicable to the second stage of our inference engine, in which we should eliminate derivations not satisfying certain meta norms.

III A Logic For Legal Reasoning

The logic of reasonable inferences with which we propose to model legal reasoning will use the language of predicate calculus, as this language seems powerful enough to express legal rules and factual situations, without loosing any relevant information. Section 3.1 lists our notational conventions and illustrates some predicate calculus concepts. In section 3.2 the logic of reasonable inferences is defined, and section 3.3 compares the logic of reasonable inferences with Reiter's Default Reasoning and with Poole's framework for Default Reasoning.

3.1 Predicate Calculus Conventions

This section describes those concepts of predicate calculus we use to define the logic. It is intended to be a quick reminder, not to be an exhaustive or precise introduction. We therefore presuppose some elementary knowledge of predicate calculus, and assume the more rigorous definitions will be used for the concepts we will only mention here.

$\mathcal{L}$ is the language of the logic containing all syntactically correct formulas, called the *well-formed formulas* (*wffs*) of the logic. It will contain predicate symbols such as =, function symbols, and logic operators such as $\wedge$ (and) , $\vee$ (or) and $\rightarrow$ (implication).

A *theory* Γ is a set of wffs in $\mathcal{L}$. The *semantic derivability relation* denoted by $\models$ makes the distinction between correct and incorrect conclusions drawn from a theory. If a wff φ is semantically derivable from a theory Γ, we write $\Gamma \models \varphi$. The definition of $\models$ is the usual one.

If Γ is empty we write $\models \varphi$, which means that φ is universally valid. A theory Γ is called inconsistent if there exists some φ such that both $\Gamma \models \varphi$ and $\Gamma \models \neg\varphi$ hold. A theory is called consistent if and only if (*iff*) it is not inconsistent.

3.2 Inconsistency And The Logic Of Reasonable Inferences

The predicate calculus definition of semantical derivability seems to be a pretty reasonable one, but it enjoys a peculiar property if theories are allowed to be inconsistent: anything can then be derived from them! Thus, if Γ is an inconsistent theory, then $\Gamma \models \varphi$ for any $\varphi \in \mathcal{L}$. Theories like this are called *trivial*, and logics that render inconsistent theories trivial are called *explosive*.

Explosiveness conflicts with any intuitive understanding of derivability. We surely do not want to conclude from an inconsistent theory on environmental law that the obligation to possess an environmental permit implies that one does not perform activities which concern the environment, or that all farmers are civil servants. One is not liable to accept any derivation of a formula containing concepts not present in the theory it was derived from.

To describe this issue in a more formal framework, let Γ be an inconsistent theory in $\mathcal{L}$. Let α be a wff in $\mathcal{L}$ only containing variable, constant, predicate and function symbols that occur in some wffs in Γ, and let β be a wff in $\mathcal{L}$ containing some variable, constant, predicate and function symbols **not** occuring in any wff in Γ. Then the intuitively undesirable property can be formally described by the observation that predicate calculus with its definition of $\models$ yields $\Gamma \models \alpha$ and $\Gamma \models \beta$ for any α and β defined as above, whereas one would more or less agree with a definition of $\models$ satisfying the constraint $\Gamma \not\models \beta$ for any β as defined above (unless of course $\models \beta$ holds, in which case β is a universally valid formula).

Inconsistent theories, which model the body of rules of law, have their use in legal reasoning, as has been argued in section 1. Therefore, our definition of semantical derivability must surely avoid the property of predicate calculus derivability concerning inconsistent theories by responding to inconsistent theories along the lines described in the previous paragraph. This can be achieved by demanding that every justification for a derived conclusion is internally consistent, where a justification is the set of rules and observations (facts) used to derive the conclusion. This demand is a straightforward observation taken from legal reasoning theory ([Cro77], [Nie76]).

These constraints lead to the definition of a new semantical derivability relation $\models_r$ for the logic of reasonable inferences. The language of this logic equals that of predicate calculus.

Definition(domain of rules)

A domain of rules over $\mathcal{L}$, or *reasonable theory*, is a tuple Δ defined as

$$\Delta = \langle A, H \rangle$$

where ***A*** and ***H*** are sets of wffs in $\mathcal{L}$. ***A*** contains the *axioms*, and ***H*** contains the *assumptions* (hypotheses). ***A*** is required to be consistent.

Δ

The *assumptions* model the rules of law that may or may not be applied in some factual situation to derive a conclusion, and contain all normative or subjective classifications of the factual situation. The *axioms* are intended to be valid in every justification, and thus restrict the number of possible justifications. These axioms represent the ascertained facts and previously ascertained conclusions (the permanent database in any implementation).

Definition(position within a domain)

A *position* (or *conviction*) ϕ within a domain of rules $\Delta = \langle A, H \rangle$ is the set (or normal predicate calculus theory) defined as

$$\phi = A \cup H',$$

where ***H'*** ⊆ ***H*** and ϕ must be consistent.

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A position, then, is a set of rules taken from the domain of rules, and represents a conviction. Notice that all positions should at least contain all axioms of the domain of rules. A position is consistent by definition.

Definition(reasonable inference)

Let Δ be a domain of rules. Define a new semantic derivability-relation $\models_r$ as :

$$\Delta \models_r \varphi$$

iff there exists a position ϕ within Δ which satisfies

$$\phi \models \varphi$$

where $\models$ is the normal predicate calculus semantic derivability relation. If $\Delta \models_r \varphi$ holds, **φ** is said to be a *reasonable inference* from the **domain of rules** Δ.

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We can paraphrase this definition by stating that a wff can reasonably be inferred from an inconsistent set of wffs iff it is derivable (in the normal predicate calculus sense) from a consistent subset of this set which contains at least the axioms. Note that if a domain of rules $\Delta = \langle A, H \rangle$ is consistent (i.e. if $A \cup H = \Gamma$ is consistent), then

$\Delta \models_r \varphi \leftrightarrow \Gamma \models \varphi$. $\models_r$ behaves exactly like $\models$ if applied to consistent theories.

In this setting a **justification** for a conclusion φ derived from a domain of rules Δ is a minimal position (with respect to set-inclusion) J within Δ such that $J \models \varphi$. This definition is based on the more intuitive definition as a set of rules and statements about the factual situation used to draw the conclusion. Notice that a justification need not be unique, but is always consistent, thus satisfying our constraints.

A **context** in Δ is the union of n simultaneously derived conclusions ψ_i and their justifications J_i derived from Δ, i.e. a context is the set of tuples $\{\langle \psi_i, J_i \rangle \mid 1 \leq i \leq n\}$. The J_i must however satisfy:

$$\bigcup_{i=1}^{n} J_i \text{ is consistent}$$

This guarantees that simultaneously derived conclusions are not based on mutually inconsistent positions, and that

$$\Delta \models_r \psi_1 \wedge ... \wedge \psi_n$$

holds. (For a proof see the weak adjunction lemma stated below).

To clarify the behaviour of $\models_r$ we will consider an example.

Example 1:

Suppose we have the following domain of rules $\Delta = \langle A, H \rangle$ defined as

A = { *WCP(A), USE(A)* }

and

H = { $\forall x(USE(x) \rightarrow \neg WAS(x)), \forall x(WCP(x) \rightarrow WAS(x))$ }

From this formal structure we can derive whether *WAS(A)* or ¬*WAS(A)*, using the definition of reasonable inferences. This definition suggests that we should first of all find all possible positions within Δ. Using the definition of a position within a domain, we obtain the following positions:

$\Phi_1 =$ { $WCP(A), USE(A), \forall x(WCP(x) \rightarrow WAS(x))$ }

$\Phi_2 =$ { $WCP(A), USE(A), \forall x(USE(x) \rightarrow \neg WAS(x))$ }

Of course, all subsets of the above positions are also positions within Δ. These positions represent the possible ways (views) to tackle this legal problem. From these positions we derive the contexts:

(1) $\Phi_1 \vdash WAS(A)$

with justification: { $USE(A), WCP(A), \forall x(WCP(x) \rightarrow WAS(x))$ }

(2) $\Phi_2 \vdash \neg WAS(A)$

with justification: { $WCP(A), USE(A), \forall x(USE(x) \rightarrow \neg WAS(x))$ }

From this we can conclude $\Delta \models_r WAS(A)$ as well as $\Delta \models_r \neg WAS(A)$. This result implies that further investigation of the justifications on which these contradictory conclusions are based must resolve whether *WAS(A)* or ¬*WAS(A)* must be concluded.

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To demonstrate the viability of this logic formally, we will prove two lemmas stating important properties.

The first lemma states that the logic is safe in the sense that one can not derive contradictions from it, thus representing the property that all contexts are contradiction free.

Lemma (contradictions):

Let φ be a wff in $\mathscr{L}$. If φ is a contradiction, i.e. $\vdash \neg\varphi$, then

$$\Delta \not\models_r \varphi$$

for any domain of rules Δ.

Δ

Proof:

Suppose that φ is a contradiction, and that $\Delta \models_r \varphi$ does hold. Then there exists a position Φ within Δ which justifies φ, i.e. such that $\Phi \models \varphi$. (If $\Delta = \langle\{\},\{\}\rangle$ then $\Phi = \{\}$ will suffice.) But also $\Phi \models \neg\varphi$, since $\models \neg\varphi$, which contradicts the fact that by definition Φ is consistent.

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The next lemma states that a *weak* adjunction rule holds, if both conclusions are derived from mutually consistent justifications. This indicates that the adjunction rule only holds for mutually consistent justifications, and not for mutually inconsistent justifications. The legal connotation of this lemma is that justifications which consist of conflicting opinions can not be joined.

Lemma (weak adjunction):

Let α and β be wff's in $\mathcal{L}$. Let Δ be an arbitrary domain of rules over $\mathcal{L}$. Suppose that $\Delta \models_r \alpha$ and $\Delta \models_r \beta$. Then

$$\Delta \models_r \alpha \wedge \beta$$

iff there exist positions A and B within Δ such that

$$A \models \alpha \wedge B \models \beta$$

and $A \cup B$ is consistent.

Δ

Proof:

If $\Delta \models_r \alpha \wedge \beta$, then there exists a position Φ in Δ such that $\Phi \models \alpha \wedge \beta$, implying that $\Phi \models \alpha$ and $\Phi \models \beta$. Since Φ is a position in Δ we also get $\Delta \models_r \alpha$ and $\Delta \models_r \beta$. For the iff-part of the proof, note that since $A \models \alpha$ and $B \models \beta$ we have $A \cup B \models \alpha$ and $A \cup B \models \beta$. From this we may conclude
$A \cup B \models \alpha \wedge \beta$. Since $A \cup B$ is consistent, and both A and B are positions within Δ, $A \cup B$ is a position in Δ. This yields $\Delta \models_r \alpha \wedge \beta$.

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To show that the general adjunction rule does not hold, i.e. that it is not the case that if $\Delta \models_r \alpha$ and $\Delta \models_r \beta$ we can conclude $\Delta \models_r \alpha \wedge \beta$, the following example should suffice.

Example 2:

Let Δ be the following domain of rules $\Delta = \langle\{\},\{\gamma, \neg\gamma, \gamma \rightarrow \alpha, \neg\gamma \rightarrow \beta\}\rangle$ with suitable α, β and γ, then we have $\Delta \models_r \alpha$, with $\Phi = \{\gamma, \gamma \rightarrow \alpha\}$ and $\Delta \models_r \beta$, with $\Phi = \{\neg\gamma, \neg\gamma \rightarrow \beta\}$ but not

$\Delta \vDash_r \alpha \wedge \beta$.

Δ

This behaviour is caused by the mutual inconsistency of the justifications on which the conclusions are based.

3.3 Comparison To Default Reasoning

The logic of reasonable inferences is, in some important aspects, similar to Reiter's Default Reasoning when applied to normal default theories. There is however a crucial formal difference, and a difference in proposed use. In this section, we investigate these similarities, and indicate the points in which the two differ. For a thorough description of default reasoning, we refer to Reiter's original article [Rei80].

Default reasoning was proposed as a model for reasoning with incomplete knowledge (e.g. birds can fly, ostriches are birds) and the retraction of previously derived conclusions (e.g. ostriches can fly) in the light of new information (e.g. ostriches can't fly). For this purpose *general default rules*

$$\frac{\alpha(\vec{x}) : \mathrm{M}\ \beta_1(\vec{x}), \ldots, \mathrm{M}\ \beta_n(\vec{x})}{w(\vec{x})}$$

are introduced, with the following interpretation: ``if $\alpha(\vec{x})$ holds, then in the absence of any information contradicting $\beta_i(\vec{x})$ for any $i \in \{1, \ldots, n\}$ infer $w(\vec{x})$ ''. A default rule is called *normal* iff it has the following form:

$$\frac{\alpha(\vec{x}) : \mathrm{M}\ w(\vec{x})}{w(\vec{x})}$$

and *free* iff it has the following form:

$$\frac{: \mathrm{M}\ w(\vec{x})}{w(\vec{x})}$$

Default rules are not part of the logical language as such, but are to be considered as rules of inference (like modus ponens).

A *default theory* is a pair (D, W) of a set of default rules ***D*** and a set of wffs ***W***. A *normal default theory* is a default theory in which all default rules in ***D*** are normal.

The first point of comparison between the logic of reasonable inferences and default reasoning is that they are both non-monotonic logics. Let ***Th(T)*** be the set of wffs derivable from theory ***T*** within some logic, then the logic is called *monotonic* iff

$$T \subseteq T' \Rightarrow Th(T) \subseteq Th(T')$$

and *non-monotonic* otherwise. This definition can be understood to mean that by using a monotonic logic a conclusion derived from some theory remains valid if new statements are *added* to the theory.

The non-monotonic nature of the logic of reasonable inferences is stated in the next lemma (a similar lemma holds for default reasoning, see [Rei80, p.75 Theorem 3.2], and for Poole's framework, see [Poo88, p.30 Lemma 2.5]):

Lemma (semi-monotonicity):

Let $\Delta = \langle A, H \rangle$ be a domain of rules, and define Th_r (the closure under reasonable inference) by

$$Th_r(\Delta) = \{\varphi \mid \Delta \vDash_r \varphi\}$$

Then

$$(a)\ H \subseteq H' \Rightarrow Th_r(\langle A, H\rangle) \subseteq Th_r(\langle A, H'\rangle)$$

but not

$$(b)\ A \subseteq A' \Rightarrow Th_r(\langle A, H\rangle) \subseteq Th_r(\langle A', H\rangle)$$

for any *A*, *A'*, *H*, *H'*.

Δ

Proof:

To prove **(a)**, let $H \subseteq H'$. Suppose $\langle A, H\rangle \vDash_r \varphi$. Then there exists a position ϕ in $\langle A, H\rangle$ such that $\phi \vDash \varphi$. This ϕ, then, is also a position within $\langle A, H'\rangle$, yielding $\langle A, H'\rangle \vDash_r \varphi$. This proves

$$\varphi \in Th_r(\langle A, H\rangle) \Rightarrow \varphi \in Th_r(\langle A, H'\rangle)$$

and thus

$$Th_r(\langle A, H\rangle) \subseteq Th_r(\langle A, H'\rangle)$$

To contradict **(b)** we only need to observe that if ϕ is a position within $\langle A, H\rangle$ and $A \subseteq A'$, then it is not necessarily the case that $A' \subseteq \phi$, and thus ϕ might not be a position in $\langle A', H\rangle$.

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This lemma shows that the logic is monotonic in *H* but non-monotonic in *A*. As pointed out before, the axioms *A* are intended to model some ascertained facts and previously ascertained inferences. The legal connotation of this lemma is, that, if the axiom set is extended with an ascertained conclusion based on a choice of one of the alternative opinions, the number of derivations is restricted, because contexts including the alternative conclusions are not constructed anymore.

The similarity between the logic of reasonable inferences and default reasoning becomes apparent if we consider the logical framework for default reasoning suggested by Poole [Poo88] and applied to legal document assembly by Gordon [Gor89]. Poole defines a new semantical derivability relation $\Delta = \langle A, H\rangle$ which behaves like default reasoning with respect to free default theories, and which can be used to model general default theories [Poo88]. We paraphrase his definition, using our own notational conventions. Note that in Poole's framework defaults are explicit, where Reiter considers default rules as rules of inference.

Definition (Poole's semantics of default reasoning):

Let *F* and *D* be sets of wffs in the language of predicate calculus. *F* is the set of facts (like *W* in default reasoning) and *D* is the set of default rules (like *D* in default reasoning, but the default rules are now denoted as ordinary wffs). The new semantical derivability relation $\vDash_d$ is defined as

$$F, D \vDash_d \varphi$$

iff there exists a subset *D* of all possible ground instances of wffs in *D* such that $F \cup D$ is consistent and

$$F \cup D \vDash \varphi$$

where $\vDash$ is the normal semantical derivability relation of predicate calculus. (A *ground instance* of a wff ψ is the wff resulting from renaming all bound variables to unique variable names in ψ, then removing all quantifiers in ψ (thus freeing all bound variables) from the result, and substituting constants for all free variables (i.e. all variables) after that).

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Formal differences

The crucial difference between the definition of $\vDash_r$ and Poole's $\vDash_d$ is that Poole specifies that *D* is a subset of all possible **ground instances** of wffs in *D*, whereas we (if we equal *A* to *F* and *H* to *D*) specify *H'* (equals *D*) to be just a subset of *H*.

Consequently, *D* does not contain the default rules but only a subset of ground instances, whereas *H'* does contain the rules themselves. This reflects the formal difference between default rules and the formal representation of rules of law. Default rules represent general statements about reality which can be overridden by facts, rules of law represent, possibly co-existing, opinions about the normative properties of reality, which only can be overruled by applying meta rules. At a more concrete level this implies that the logic of reasonable inferences insists that *all* consequences of a rule (representing an opinion) applied once within some context should hold within that context, that is to say no opposing opinions are allowed to co-exist within one context. We clarify this by the example introduced before.

Assume that we have the following set of facts, representing one case,

$$F = A = \{WCP(rubble), WCP(wreckage), USE(rubble), USE(wreckage)\}$$

in both systems, and the following set of rules

$$D = \left\{ \frac{USE(x) : \mathrm{M} \neg WAS(x)}{\neg WAS(x)}, \frac{WCP(x) : \mathrm{M}\ WAS(x)}{WAS(x)} \right\}$$

in default reasoning and

$$H = \{\forall x (USE(x) \rightarrow \neg WAS(x)), \forall x (WCP(x) \rightarrow WAS(x))\}$$

in the Logic of Reasonable Inferences, modelling the same rules.

Then in Default Reasoning the following ground instances are produced:

1. $USE(rubble) \rightarrow \neg WAS(rubble)$
2. $USE(wreckage) \rightarrow \neg WAS(wreckage)$
3. $WCP(rubble) \rightarrow WAS(rubble)$
4. $WCP(wreckage) \rightarrow WAS(wreckage)$

Combined with the facts Default logic produces four extensions:

1. *Th(WCP(rubble), WCP(wreckage), USE(rubble), USE(wreckage), USE(rubble) → ¬WAS(rubble), USE(wreckage) → ¬WAS(wreckage),* ***¬WAS(rubble), ¬WAS(wreckage)*** *)*
2. *Th(WCP(rubble), WCP(wreckage), USE(rubble), USE(wreckage), USE(rubble) → ¬WAS(rubble), WCP(wreckage) → WAS(wreckage),* ***¬WAS(rubble), WAS(wreckage)*** *)*
3. *Th(WCP(rubble), WCP(wreckage), USE(rubble), USE(wreckage), USE(wreckage) → ¬WAS(wreckage), WCP(rubble) → WAS(rubble),* ***¬WAS(wreckage), WAS(rubble)*** *)*
4. *Th(WCP(rubble), WCP(wreckage), USE(rubble), USE(wreckage), WCP(rubble) → WAS(rubble), WCP(wreckage) → WAS(wreckage),* ***WAS(wreckage), WAS(rubble)*** *)*

whereas, the logic of reasonable inference will only produce two contexts.

The logic of reasonable inferences defines the following positions:

$$\Phi_1 = \quad \{ \mathit{WCP(rubble), USE(rubble), WCP(wreckage), USE(wreckage),} \ \forall x(\mathit{USE}(x) \rightarrow \neg \mathit{WAS}(x)) \}$$

$$\Phi_2 = \quad \{ \mathit{WCP(rubble), USE(rubble), WCP(wreckage), USE(wreckage),} \ \forall x(\mathit{USE}(x) \rightarrow \neg \mathit{WAS}(x)) \}$$

From this the following contexts will be produced:

(1) $\Phi_1 \vDash \quad \{ \neg \mathit{WAS(rubble)} \wedge \neg \mathit{WAS(wreckage)} \}$

with justification $\{\mathit{WCP(rubble),WCP(wreckage),USE(rubble),USE(wreckage),} \forall x(\mathit{USE}(x) \rightarrow \neg \mathit{WAS}(x))\}$

(2) $\Phi_2 \vDash \quad \{ \mathit{WAS(rubble)} \wedge \mathit{WAS(wreckage)} \}$

with justification $\{\mathit{WCP(rubble),WCP(wreckage),USE(rubble),USE(wreckage),} \forall x(\mathit{WCP}(x) \rightarrow \mathit{WAS}(x))\}$

These contexts are produced following the constraint that a rule, representing an opinion, once used within a context should hold within that context, thus discarding the contextual translations of extensions 2 and 3.

Pragmatic differences

A pragmatic difference between the logic of reasonable inferences and default reasoning is that the former insists that no opposing opinions are allowed within one context and that the latter allows ground instances from opposing opinions to be part of the same extension, thus implicitly tolerating opposing opinions within one solution. The logic of reasonable inferences allows for quantified formulas (containing bound variables), and not just ground instances, to be part of a position and context, thus allowing the general rule to be contained in them, as is clearly expressed by the given example. This makes specific demands on the implementation of the logic. It does not mean that default reasoning cannot be used to generate multiple extensions, but that it is not intended to represent alternative opinions by them, e.g. to compare them by using meta rules etc.

IV Implementation Of The Logic

In this section we will describe an implementation of the logic of reasonable inferences. As described in previous sections, the expert system being implemented is rule-based. The expert system contains an inference engine, a relational database system, and a graphic user interface. It has been implemented in C instead of PROLOG, thus providing us with as much control as possible over the overall efficiency of the shell (a PROLOG version and a short discussion of the related efficiency problems were given in [Vey89b]) and over its communications with external user interfaces and external databases. A PROLOG implementation would render the system too slow for practical purposes, considering the large databases (which will contain descriptions of hundreds of industries) and the large rulebases (which will contain thousands of rules from the complex domain of environmental law) needed.

The inference engine can be viewed as ``executing'' a rulebase, and to produce all possible contexts (i.e. all possible conclusions and their justifications based on the rules contained in the rulebase). Such a rulebase represents wffs in the language of the logic, but in a different syntax. To represent a domain of rules and the results one wants to derive the rulebase contains variable, rule and goal declarations.

We use predicates to represent some classification of objects in the real world (to be represented). The predicates can be connected to the database with a SQL query, in which case their value is specified by the result of that query. Therefore, the contents of the database are also part of the domain of facts one wants to specify.

The simplified syntax of a rule is:

IF <expr><logop> ... <logop><expr> **THEN** <expr> = <bindexpr>

where

<*logop*> is a logical operator, such as **AND** and **OR**.

<*expr*> is an expression evaluating as true or false. This expression may contain predicate symbols including predefined predicate symbols such as **=**, **<>**, **<** etc., and predefined function symbols such as **+**, **-**, *****, etc. It also can be a variable <*var*> if it occurs after the **THEN**.

<*var*> is a variable name

<*bindexpr*> is an expression evaluating to any predefined domain of values (integers, reals, booleans, and the like). See also <*expr*>.

The intuitive meaning of a rule can be defined in the following way: if one can derive that the condition (i.e. the expression after the **IF**) holds, one can also derive that the value of <*expr*> equals <*bindexpr*>. All predicate variables used in these rules are implicitly universally quantified.

A particular case is evaluated by supplying the problem solver with a casebase (containing the facts from the database concerning one case) and a rulebase (containing the variables, rules and goals).

The goals are predicates, the values of which one is interested in. The value of a goal is either determined by consulting the database or by evaluating an appropriate rule. During the evaluation process multiple contexts, consisting of the value of the goal and their justifications, are constructed, after which the different contexts are presented to the user. Following a choice of one context, the evaluated goals of the context are registered in the permanent database and thus turned into ascertained facts. The contexts as a whole are separately preserved to be used for explanation and meta inferencing purposes.

We now turn to the particular case as described above. For the encoding of this case, the following rulebase is used.

VARIABLES
material

RULES
IF *is_waste_in_common_parlance(x)* **THEN** *is_waste(x)=true* (1)
IF *is_used(x)* **THEN** *is_waste(x)=false* (2)

GOALS
is_waste(material)

The variable ``material'' is defined by a query which evaluates to all values contained in the casebase (in this case rubble and wreckage). Subsequently the goal is expanded to multiple goals, one for every material found in this case. In this case the goal *is_waste(material)* is expanded to: *is_waste(rubble)*, *is_waste(wreckage)*.

To find a value for the goal *is_waste(rubble)* rule (1) or rule (2) is selected, and x is unified with the constant rubble. To evaluate the predicate *is_waste_in_common_parlance(rubble)* the database is consulted using the query connected to this predicate, with rubble substituted in the query.

This results in the following facts used in executing the rulebase.

FACTS
is_used (rubble)
is_used (wreckage)
is_waste_in_common_parlance (rubble)
is_waste_in_common_parlance (wreckage)

The execution of the rulebase is concluded with the construction of two contexts, one context containing the conclusion that rubble and wreckage are waste, on the basis of the fact *is_waste_in_common_parlance(rubble)* and *is_waste_in_common_parlance(wreckage)* and the rule (1), and a second context containing the conclusion that both materials are not waste, which is justified by the facts *is_used(rubble)* and *is_used(wreckage)* and rule (2). Consistency of the contexts is guaranteed by not allowing two rules, which contain the same predicate or variable symbol as a conclusion, to be part of one context.

Note that contexts in which one material is considered waste, on the basis of rule (1), and the other material is not, on the basis of rule (2), are not constructed. This properly reflects the notions of the logic of reasonable inferences. If the formal framework as described in [Poo88] had been implemented, these contexts would have been constructed as well.

V Conclusions & Further Research

As pointed out in the introduction, our research is aimed at the development of an expert system, which can process conflicting opinions inherent in the legal system. To formally specify its inference engine, the logic of reasonable inferences was conceived. Our experiences with both the expert system and the logic itself indicate that our approach is viable and that the system satisfies the demands imposed by legal reasoning.

Legal research will be performed to write more rulebases containing the transcription of several written laws. The optimisation of the expert system will also be a considerable concern of research. [Kle86a], [Kle86b] and [Kle86c], for example, contain some interesting material on the efficiency of problem solvers and the maintenance of contexts and the dependency of computations on contexts.

At this moment, there exist no provisions to guide the user of our expert-system in the selection of an appropriate context/justification. Further research (legal research that is) is necessary to obtain existing criteria to discriminate several contexts. Computational models for discriminating contexts using meta norms will also be looked into.

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