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INCONSISTENCY AND LEGAL REASONING

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Summary

A formal analysis of a fundamental aspect of legal reasoning is proposed: dealing with normative conflicts, solving inconsistencies.
Firstly, examples are illustrated concerning the dynamics of legal systems and the application of rules and exceptions. Then, criteria to establish an ordering over legal norms are defined on the basis of the traditional principles of lex superior, lex posterior, and lex specialis. Finally, two approaches to cope with conflicting information, using an ordering, are presented: the preferred theories of Brewka, and the normative hierarchies of Alchourrón and Makinson. The two approaches are applied to legal systems, their relations are closely examined, and some aspects of a model of reasoning with normative conflicts are outlined.

1. Introduction: incompatibilities and inconsistencies in the law

Lawyers frequently have to deal with normative conflicts when applying the law: valid norms establish incompatible legal qualifications for the same concrete case. This fact has two main causes: the dynamics of legal systems, and the rule-exception formulation of the law:

- In order to cope with social and political change, law defines procedures through which norms can be produced or eliminated. The freedom of the legislator is limited only by the need to follow the legal procedures, and by the hierarchical relations among the legal sources. So, new norms can be enacted that are incompatible with norms already in force. Sometimes the legislator, while introducing a new regulation, explicitly abrogates or modifies incompatible previous prescriptions, so as to prevent conflicts. Nevertheless, not all normative conflicts are, or even can be (given the complexity of modern legal systems), prevented by the legislator.
- Law has to deal with complex social situations, in which conflicting interests are present, demanding incompatible legal solutions (for example, protecting the life and personal integrity of everybody, and providing the possibility of self defence). Moreover, the legal decision-maker often has only a limited knowledge of these situations. In such contexts, the balancing of the conflicting interests is frequently implemented through a combination of rules and exceptions. The solution established by the rule, *i.e.* corresponding to the interest normally protected by the law in a certain type of situation, can be derived (once such a situation has been ascertained in a concrete case) unless the rule is contradicted (in that case) by an exception. The exception establishes that a prevailing circumstance requires a different legal solution (even though the conditions for the application of the rule have been satisfied).

Dynamic normative systems and sets of rules and exceptions share a common aspect that justifies the attempt to treat them in a unified framework. In fact, in both

contexts a contradiction can arise as a consequence of new information (newly enacted norms, conflicting with those already in the system; and ascertained facts satisfying both the rule and the exception), so that we cannot simply add the new information, and use the classical notion of logical inference to derive legal consequences (since *ex falso sequitur quodlibet*). In both, two alternative approaches seem possible:

- To obtain consistency by eliminating or modifying some of the norms in conflict (*i.e.*, by "interpreting" the *prima facie* conflicting norms into a consistent whole).
- To develop a notion of legal consequence different from the notion of logical consequence, so that, even if the system is logically inconsistent, no inconsistent legal consequences can be derived from it.

The second solution has a number of advantages, in representing legal knowledge and in modelling legal reasoning. Its choice is supported by the following considerations:

- In most cases all conflicting norms are to be considered valid. The lawyer uses an ordering of the norms involved to solve the conflicts and determine consistent legal consequences: the weaker norms continue to be applied, but only by default, *i.e.* unless stronger incompatible norms are applicable in the same case.
- If some of the conflicting information is later eliminated from the system[1], so that no inconsistency arises any longer, the weaker norms can become applicable again.

The extent of the application area of the conflicting regulations cannot be assessed independently of the conflicts handling procedure[2]. The attempts to obtain, through "interpretation", a consistent reformulation of the legal system seem just the *a posteriori* rationalization (insufficient in explanatory power, as we shall see) of a more basic aspect of legal reasoning[3].

The derivation of the legal consequences of an inconsistent legal system is not reducible to logical inference, but, nevertheless, can be modelled by means of logical notions. So, in the following pages, a non logical notion of legal consequence will be developed by relating two different conceptual frameworks, both logic oriented: the preferred theories approach, proposed by Brewka (1991) for nonmonotonic reasoning, and the treatment of normative hierarchies developed by Alchourrón and Makinson (1991).

We shall now introduce some concepts for later use.

Let a *legal system* be a set of (legal) norms, a *case* a set of facts[4], a *legal qualification system* the union of a legal system and a case.

Let us introduce a logical consequence operation *Cn*[5], and let us write *Cn*(*A*) for the deductive closure of *A*, and $A \mid p$ for $p \in Cn(A)$. If $A \mid p$ we say that *p* is a *logical consequence* of *A*, or that *p* is *logically derivable* from *A*.

Let us call a legal systems *A inconsistent* iff $A \mid \boldsymbol{F}$, where $\boldsymbol{F}$ denotes contradiction. Let us call a legal system *A incompatible* iff *A* can lead to contradiction in some case, *i.e.* iff there is a possible (consistent) case *C* such that the legal qualification system $A \cup C$ is inconsistent.

Let us call *theory* a set of statement closed under *Cn*. We will use the expression *base*, or *theory* (not underlined) to designate any set of statements.

We adopt the convention to assign names to norms, that will be used to express cross references. The name of a norm is a predicate, followed by the free variables in the norm[6] (by replacing variable names with constants, we have names for all ground instances of the same norm)[7]. So, a norm will be expressed as follows:

name(***x***): *conclusion* ← *condition*

where ***x*** is the tuple of the free variables included in *conclusion* and *condition*. In all our examples, the conclusion is a literal and the condition a conjunction of literals.

2. Two examples

To make our discussion more concrete, we will first introduce two examples, one concerning the legal dynamics, the other, the application of rules and exceptions.

2.1. Reasoning within a dynamic legal system

In the Italian legal system there is a general principle stating contractual liberty: the parties are free to determine the content of their contracts, to establish whatever contractual relations they like[8]. Let us represent this general rule simply by:

art_1322_CC(*LegalRelation*, *Act*):
holds(*LegalRelation*) ←
    happens(*Act*) *contract*(*Act*)
    establishes(*Act*, *LegalRelation*)

meaning that a certain legal relation holds if it is established by a contract.
In the Seventies, strict limitations to contractual freedom were introduced in tenancy law. In particular, Art 12 State Act No 329, of 1978, stated that rent prices could not be freely established by the parties, but were determined by fixed legal criteria. The contractual determination of a different rent price was to be considered void (not holding).

art_12_329/1978(*RentPrice*, *House*, *LegalPrice*):
¬*holds*(*rent_price_of*(*RentPrice*, *House*)) ←
    habitation(*House*) *legal_rent_price_of*(*House*, *LegalPrice*)
    RentPrice ≠ *LegalPrice*[9]

As a result of the enactment of this norm, prevailing over the general principle of contractual liberty by both the criteria of *lex posterior* and *lex specialis* (**art_1322_CC** <**art_12_329/1978**)[10], that principle could not be applied to the determination of the rent price for dwellings (but it could be applied to other aspects of tenancy contracts, as to other types of contracts).
In the Eighties, the political climate changed (deregulation became the new slogan), and new norms where introduced, as exceptions excluding, in certain parts of the Italian territory, the constraints on rent prices. Here is the formalization of an example of one of such exceptions.

art_12_329/1978_exc$_1$(*RentPrice*, *House*, *LegalPrice*, *Municipality*, *InhabNum*):
¬*art_12_LS_329/1978*(*RentPrice*, *House*, *LegalPrice*)[11] ←
    situated(*House*, *Municipality*)
    inhabitants_of(*Municipality*, *InhabNum*) *InhabNum* < 20.000.

The constraint on rent prices is blocked by this last exception (assuming that **art_12_329/1978** < **art_12_329/1978_exc$_1$**), so that the general principle of contractual liberty expands, again regulating the determination of rent prices in the municipalities with less than 20.000 inhabitants[12].
Now, it seems that the Italian Parliament intends to explicitly abrogate Art. 12 State Act 329/1978. As a consequence, art 1438 Civil Code would recover its full applicability in the rent price domain.
The example shows that preserving all conflicting norms in a dynamic legal system has some important advantages:

Let us assume the following ordering:

parent_liability < ***parent_liability_exc*$_1$**;
fault_liability < ***incapacity***;
fault_liability < ***self_defence***;
incapacity < ***incapacity_exc*$_1$**.

The resulting legal system is consistent but incompatible. Let us consider what happens by adding convenient facts (so as to obtain a legal qualification system). Let us assume, for example, that Mary wants compensation from Mark, father of John, saying that John, who was invited to a party in her house, has broken her precious Chinese vase. She has been able to prove the following facts (we name facts with numbers, and assume that they are stronger than any norm, accordingly to the < relation):

1: *accomplished(john, chinese_vase_push).*
2: *caused*(*chinese_vase_push, chinese_vase_distruction*).
3: *unlawful*(*chinese_vase_distruction*).
4: *at_fault_for*(*john, chinese_vase_push*).
5: *parent*(*mark, john*).

Given these facts, consistency is preserved, and both Mark and John are liable for the destruction of the vase (since the conditions of the liability norms are satisfied and the condition of no exception is).
Mark can free himself by satisfying the condition of one of the exceptions, so to cause a contradiction with the rules establishing liability. So, for example, if he proves that John was incapable at the moment of the accident, *i.e.* that

6: *incapable_during*(*john, chinese_vase_push*),

John's liability is excluded by the exception ***incapacity***, whose consequent contradicts the consequent of the rule ***fault_liability***.
But Mary can prove that John's incapability derived from his fault, since it was caused by his drunkenness. So the exception ***incapacity*** is denied (its applicability is excluded) by the stronger exception ***incapacity_exc*$_1$**, John appears liable again, and therefore Mark.
Then Mark could try to prove that he could not control his son, so as to prevent the fact, that is

7: ¬*could_prevent*(*mark, chinese_vase_push*),

and free himself again from the liability, through a new contradiction (between the rule ***parent_liability*** and the exception ***parent_liability_exc*$_1$**).
In every phase of the dispute, contradictions among rules and exceptions assume a particular relevance: if the condition of an exception denying a rule or its effect is proved, the derivation of the consequent of the rule is blocked.

It would have been possible to prevent inconsistencies, by rewriting any norm so that its condition would include the complement of a literal occurring in the body of any exception to the norm itself or to its effect (exceptions can, then, be eliminated)[16]:

liable_for(*Par*, *Damage*, *Fact*) ←
parent(*Par*, *Son*) *liable_for*(*Son*, *Damage*, *Fact*)
¬*could_prevent*(*Par*, *Fact*).

liable_for(*X*, *Damage*, *Fact*) ←
accomplishes(*X*, *Fact*) *caused*(*Fact*, *Damage*)
wrongful(*Damage*) *at_fault_for*(*X*, *Fact*)
¬*did_by_self_defence*(*X*, *Fact*) ¬*incapable_during*(*X*, *Fact*).

liable_for(*X*, *Damage*, *Fact*) ←
accomplishes(*X*, *Fact*) *caused*(*Fact*, *Damage*)
wrongful(*Damage*)
at_fault_for(*X*, *Fact*) ¬*did_by_self_defence*(*X*, *Fact*)
incapable_during(*X*, *Fact*) ¬*incapable_by_fault_during*(*X*, *Fact*).

This "interpretation" is, not only contrary to our intuitions, but also not equivalent to the above set of conflicting rules and exceptions. The main differences are the following:

- Normative systems including rules and exceptions allow nonmonotonic reasoning: the conclusions of a rule can be drawn, provided that the conclusion of no exception (to that rule or to its conclusion) is derivable. This means that the legal decision-maker (e.g., the judge) must derive the conclusion of the rule not only when he has positively ascertained that none the exceptions (to that rule or to its conclusion) are satisfied, but also when he has no information about them. If further information is added, allowing the consequent of the exception to be derived, the consequent of the rule must be retracted. In the consistent "interpretation", this aspect is lost.
- Rules and exceptions have a dynamic aspect: if a new rule is enacted, exceptions automatically apply to it[17], and if an exception is abrogated the rules expand themselves, *i.e.* are applicable in cases where they were previously blocked by contradictions with the exception[18]. We cannot obtain the same effect by adding or removing statements to the "interpretation".

3. The hierarchy of legal norms

In the previous examples it has been shown that reasoning with inconsistency in the law relies on a hierarchical ordering established over the legal system. Let us examine how this ordering is defined.
For this purpose, the traditional principles regulating the relation between norms must be considered: the *source* criterion (*lex superior*), the *chronological* criterion (*lex posterior*), and the *speciality* criterion (*lex specialis*). The corresponding relations will be denoted by $<_{SR}$, $<_{T}$, and $<_{S,}$ respectively:

- Source criterion. Any norm n_2 issued by (belonging to) a higher source of law is preferred, according to the source criterion, to any norm n_1 issued by a lower source[19], to wit $n_1 <_R n_2$.
- Chronological criterion. Any norm n_2 issued by a later normative act is preferred, according to the chronological criterion, to any norms n_1 issued by a preceding normative act, to wit $n_1 <_T n_2$.
- Speciality criterion. Any more special norm n_2 is preferred, according to the speciality criterion, to any more general norm n_1, to wit $n_1 <_S n_2$[20].

As a rule, an ordering over the criteria (a "metaordering") is also assumed: the source criterion prevails over both the speciality and the chronological ones, while the speciality criterion prevails over the chronological one.
The three relations $<_{SR}$, $<_T$, and $<_S$ are irreflexive and transitive. Therefore, given the priorities just described among the criteria, it is not difficult to combine them into a ordering relation < over a legal system.
A norm n_2 prevails over a norm n_1 ($n_1 < n_2$) iff

1. $n_1 <_{SR} n_2$;
2. $n_1 <_S n_2$, provided that $n_2 \not<_{SR} n_1$;
3. $n_1 <_T n_2$, provided that $n_2 \not<_{SR} n_1$ and $n_2 \not<_S n_1$.

We also have to consider norms that are equivalent as far as the three criteria are concerned, and denote the equivalence by source, by time, and by speciality, as $\equiv_{SR}$, $\equiv_T$, and $\equiv_S$, respectively. All sources and time instants must be comparable[21], the same cannot be assumed for speciality. So, we can define two norms as equivalent iff they are equivalent by source and time, and none of them is prevailing by speciality:

$n_1 \equiv n_2$ iff $n_1 \equiv_{SR} n_2$ and $n_1 \equiv_T n_2$, provided that $n_2 \not<_S n_1$ and $n_1 \not<_S n_2$.

Finally, we can say that

$n_1 \leq n_2$ iff $n_1 < n_2$ or $n_1 \equiv n_2$.

Extending the ordering $\leq$[22] over the legal system A to a hierarchy over the legal qualification system $A \cup C$ (including also the facts of a case C) is quite simple. It is sufficient to assume that facts are preferred to any norm, i.e. that for any $p \in A$ and any $q \in C$, $p < q$.
It may seem strange that facts of the case are preferred to every legal norm. In fact, conflicts between just one norm and a (consistent) case are excluded, under reasonable conditions[23]. An inconsistency can result only when the legal qualification system includes more than one norm. But in such a situation we ascribe to the norms, not to the facts, the responsibility of the conflict, and the conflict is solved by rejecting one of the norms. So, a conflict between rule and exception can only happen when the facts satisfy the conditions of both of them. But it would not be reasonable to hold the facts as responsible for the conflict and to solve it by rejecting one of them.

4. The preferred theories framework

To give a formal account of the aspects of legal reasoning outlined above, we will consider first the preferred theories framework proposed by Brewka (1991), an approach generalizing abduction, and inspired by Rescher (1964).
Brewka proposes to define the notion of provability from a possibly inconsistent set of premises (a theory) *T*, by taking into consideration just the preferred maximal consistent subsets of *T*, *i.e.* its *preferred subtheories*[24]. So, a formula p is defined as *weakly probable* from a set of premises T iff there is a preferred subtheory B | T such that $B \mid p$. A formula p is defined as *strongly provable* from *T*, iff for all preferred subtheories B of *T*, $B \mid p$ (Brewka 1991, 65).
In this framework two notions of legal consequence are definable: in a weak sense, the legal consequences of a legal (or legal qualification) system A are the formulae weakly

provable from *A*, *i.e.* derivable from a preferred subtheory *B* of *A*; in a strong sense, the legal consequences of *A* are the formulae strongly provable from *A*, *i.e.* derivable from all preferred subtheories.
Brewka (1991, 65) considers level default theories. A level default theory is tuple $(T_n, \ldots, T_1)$, obtained by partitioning a theory (*i.e.*, a base) *T* into a hierarchy of levels $T_n, \ldots, T_1$, where each T_i is a set of classical first order formulae.

T_n	Hypotheses
T_{n-1}	Hypotheses
T_2	Hypotheses
T_3	Hypotheses

Different levels represent different degrees of reliability. The highest level is the most reliable[25]. The following (from Brewka 1991, 72) is the definition of preferred subtheory of a level default theory:

Let $T = (T_n, ..., T_1)$ be a level default theory. $S = S_n \ldots S_1$ is a preferred subtheory of *T* iff, for all *k* $(n \geq k \geq 1)$, $S_n \ldots S_k$ is a maximal consistent subset of $T_n \ldots T_k$ (Brewka 1991, 65).

More intuitively, we can say that "to obtain a preferred subtheory of *T* we have to start with an arbitrary maximal consistent subset of T_n, add as many formulae from T_{n-1} as can be consistently added, in any possible way, and continue to do so for $T_{n-2}, \ldots, T_1$" (Brewka 1991, 72)[26]. A level default theory *T* can have more than one preferred subtheory, if it includes incompatible hypotheses of the same level. Then, some formulas will be only weakly provable, that is derivable from some, but not from all, preferred subtheories of *T*.
This model can capture some aspects of legal systems. For example, we could assume that each level contains just the norms issued by a certain normative act, and that the assignment of the levels is based on the hierarchy of the legal sources and on the chronological ordering of the normative acts. Each normative act can be represented by the level $T_{i,j}$, where *i* is the position (in the source hierarchy) of the source to which the act belongs, and *j* is the chronological position of the act in relation to the other acts in the same source. Each legal source is represented by a slice of the level default theory:

T_n	Facts of the case
$T_{n-1,m}$	Last issued Constitutional Act
	…
$T_{n-1,1}$	First Constitutional Act
$T_{n-2,k}$	Last issued Parliamentary Act
	…
$T_{n-2,1}$	First issued Parliamentary Act
$T_{n-3,h}$	Last issued Government Regultaion
	…
$T_{n-3,1}$	First issued Government Regultaion

.... And so	on, until level 1

Adopting this representation, we have different subtheories just in case we have inconsistencies in the same normative act. In fact, if all norms included in the same act are compatible, and just the source and chronological orderings are considered, there is only one preferred subtheory. It includes all facts of the case, all statements in the last Constitutional Act, plus the statement in the previous Constitutional Act consistent with the statements already in the preferred subtheory, ... , plus the norms in the last ordinary Act of Parliament consistent with the facts and Constitutional norms already in the preferred subtheory, and so on.
The legal consequences of the inconsistent normative system are exactly the logical consequences of its preferred subtheory.
This approach accounts for situations as described in the example of section 2.1, where a subsequent legislation made certain legal consequences no longer derivable, and the abrogation of a norm caused the "expansion" of a previously issued provision (as for the principle of contractual liberty).
On the contrary, it is unable to cope with the rules and exceptions from the Italian tort law of section 2.2., where preference relations exist between norms included in the same normative act (the Italian Civil Code).
In order to develop a general solution, we cannot simply modify the default level theory by introducing intermediate levels. In fact the model of level default theories compels us to assign a level to every statement, that is to establish that every norm is in a precise relation to every other norm: it has the same priority as the norms included in the same level, a lower priority than the norm included in higher levels, and a higher priority than norms included in lower levels. This cannot always be done in law, since, the speciality ordering concerns just certain norms. Moreover, the speciality criterion prevails over the chronological one, so that it can subvert the chronological ordering.
A generalization overcoming this problem is obtained (cf. Brewka 1991, 75) by considering a strict partial ordering $>$[27] on a finite set of premises T[28]. The preferred subtheories are built by considering all possible linearizations of the premises in T.

Let $>$ be a strict partial ordering on a finite set of premises T. S is a preferred subtheory of T iff there exists a strict total ordering $(t_n, t_{n-1}, \ldots, t_1)$ of T respecting $>$ (*i.e.* $t_k > t_j$ $k > j$) such that $S = S_0$ with
$S_{n+1} := \{\}$, and for $n + 1 \geq i > 0$.
$S_{i-1} :=$ if t_{i-1} consistent with S_i, than $S_i \cup \{t_{i-1}\}$, else S_i.

Defining $>$ as we did in section above, we can give an account of legal reasoning in a hierarchy of norms whose ordering is constructed by combining the three criteria there introduced.
The strong (certain) legal consequences are those derivable from all preferred subtheories, the weak (dubious) ones, are those derivable just from some of them.
In fact, multiple preferred subtheories are obtained just when we have a real uncertainty in the law, *i.e* when the legal system contains norms n_1 and n_2 that are in conflict (such that both can individually be consistently added to the subtheory, but whose joint insertion would cause an inconsistency), and between which the preference relation $>$ does not hold ($n_1 \not> n_2$, and $n_2 \not> n_1$).

5. The proposal of Alchourrón and Makinson

In this section we will develop a different definition of the preferred subtheories (we will speak, more generally, of preferred subsets), leading to the same results as Brewka's last definition.
For this purpose we need to transform the relation ≤ defined over the norms in the legal system *A*, into a relation ≤ defined over the subsets of *A*. Our starting point is the approach to reason with hierarchies of legal norms developed by Alchourrón and Makinson (1981).
These authors propose to solve contradictions in the application of the law by taking into consideration an ordering ≤ over the legal system. A judge can deliver a justified verdict on the basis of an incompatible normative set by using (and eventually extending) that ordering. To illustrate this aspect of legal reasoning Alchourrón and Makinson introduce the following notions:

If (*A*, ≤) is a hierarchy of regulations and *B*, *C* *A* we shall say that *C* is at least as exposed as *B*, and write *C* ≤ *B*, iff for every *b* *B* there is a *c* *C* with *c* ≤ *b*. … If *B*, *C* *A* we shall say that *C* is strictly more exposed than *B*, and write *C* < *B*, iff *C* ≠ Ø and for all *b* *B* there is a *c* *C* with *c* < *b* (Alchourrón and Makinson 1981, 126-127).
We say that *B* *indicates* *p* and write ≤: *B* → *p* iff *p* *Cn*(*B*), and moreover for all *C* *A*, if ¬*p* *Cn*(*C*), then *B* ò *C*. We say that *B* *determines* *p* and write <: *B* → *p* iff *p* *Cn*(*B*), and moreover, for all *C* *A*, if ¬ *p* *Cn*(*C*), then *C* < *B* ….
We say that a hierarchy (*A*, ≤) *delivers* a proposition *p*, and write <: *A* *p*, iff some subset *B* *A* determines *p* …
For each *a*, we say that *a* is *normal* iff, for every inconsistent *C* *A*, there is a *c* *C* with *c* < *a* (Alchourrón and Makinson 1981, 136-138).

Alchourrón and Makinson show that the delivered propositions are those derivable from the set *N* of the normal elements of *A*.
It is easy to see (cf. Alchourrón and Makinson 1981, 138 f) that the set *Cn*(*N*) of the consequences of *N* does not contain all the intuitive legal consequences of *A*. Therefore, the concept of delivering does not give a satisfying notion of legal consequence. In fact, if *A* contains both *p* and *q*, and {*p* , *q*} is inconsistent, then no axiom *r* such that *r* < *p* and *r* < *q* can be in *N*, to wit *r* cannot be used in delivering any proposition from *A*. So, if an unconstitutional Parliamentary Act is passed, no decision grounded on administrative regulations can be delivered.
To obtain a satisfying notion of legal consequence, let us redefine the preference relation between the subsets of the normative system *A*.
Let us say that two sets *X* and *Y* are *conflicting* on a formula *p* iff *X* | *p* and *Y* | ¬*p*. Let us say that *Y* is *prevailing* over *X* on *p*, iff *X* and *Y* are conflicting on *p*, and for all *X* ′ *X* conflicting with *Y* on *p* there exists a *Y* ′ *Y* such that *X* ′ and *Y* ′ are conflicting on a formula *q* and *X* ′ < *Y* ′. Finally, set us say that *Y* is *preferred at least as* *X*, and write *X* ≤ *Y*, iff there exist no formula *p* such that *X* is prevailing over *Y* on *p*. Moreover, we say that *Y* is *preferred to X*, and write *X* < *Y*, iff *X* ≤ *Y* and *Y* n *X*, *i.e.* iff there exists no *p* such that *X* prevails over *Y* on *p*, but there exists a *q* such that *Y* prevails over *X* on *q*. Intuitively, the definition of preferability states that if the preferred

set Y conflicts with X, then there must be a $Y' \subseteq Y$, better (over $<$) than any $X' \subseteq X$ conflicting with Y, that supports Y. The relation $\leq$ is transitive, when X and Y are maximal consistent subsets of a given set A.

Let us formalize the notions of indicating and determining on the basis of the $\leq$ relation.

We can say that $B' \subseteq A$ *indicates* a verdict p, iff $p \in Cn(B')$, $B' \subseteq B$ and B is a maximal (over $\subseteq$ and $\leq$) consistent subset of A. This means that p is weakly provable.

We say, that $B' \subseteq A$ *determines* a verdict p, iff $p \in Cn(B')$, and for all maximal (over $\subseteq$ and $\leq$) consistent subsets $B \subseteq A$, $B' \subseteq B$. This means that B' can be used as a justification of p in every B, *i.e.* that B' is included in (and p is implied by) the intersection of the Bs. Determination is stronger than strong provability, since (when the set A is not a theory) we can have statements derivable from all Bs, but not from their intersection.

6. Notions of legal consequence

Brewka's and Alchourrón and Makinson's proposals (the latter integrated by our definition of the $\leq$ preference relation) have allowed us to identify a similar solution to the problem of dealing with an inconsistent set of legal premises.

In both approaches, a family of preferred maximal consistent subsets of the legal, or legal qualification, system A under consideration is obtained[29].

Once such a family of preferred subsets is identified, at least four interesting concepts can be defined:

a. *Weak legal consequence*. A statement p is a weak legal consequence of A iff it is derivable from a preferred maximal subset of A. This notion corresponds to Brewka's weak provability, and to our formalization of Alchourrón and Makinson's indicating. If we denote as $P_R(A)$ the set of preferred maximal consistent subsets of A, then the set $W_{LC}(A)$ of the weak legal consequences of A is the union of the consequences of all $X \in P_R(A)$, to wit $W_{LC}(A) = \cup\{Cn(X : X \in P_R(A)\})$.

b. *Strong legal consequence*. A statement p is a strong legal consequence of A iff it is deducible from all the preferred maximal subsets of A. This notion corresponds to Brewka's strong provability. The set $S_{LC}(A)$ of the strong legal consequences of A is the intersection of the consequences of all $X \in P_R(A)$, to wit $S_{LC}(A) = \cap\{Cn(X : X \in P_R(A))\}$.

c. *Dubious, or uncertain, legal consequence.* A statement p is a dubious legal consequence of A iff it is deducible from some, but not from all the preferred maximal subsets of A. The set of the dubious legal consequences, that we can denote as $D_{LC}(A)$, is simply the difference of the weak and the strong legal consequences, to wit $D_{LC}(A) = W_{LC}(A) - S_{LC}(A)$.

d. *Strongly justified legal consequence*. A statement p is a strongly justified legal consequence of A iff it is deducible from the intersection of the preferred maximal subsets of A. This notion corresponds to our interpretation of Alchourrón and Makinson's concept of determining. The set $S_{JLC}(A)$ of the strongly justified legal consequences of A is the deductive closure of the intersection of all $X \in Pr(A)$, to wit $S_{LC}(A) = Cn(\cap\{X : X \in P_R(A)\})$.

Both notions of strong legal consequence, and of strongly justified legal consequence have an intuitive background. The first expresses the opportunity to minimize the choices involved in legal decisions: if all maximal preferred subsets of A allow the derivation of a certain conclusion p, this should be enough, in order to accept p as sufficiently justified. The second expresses the necessity that the legal system be coherent, *i.e.* able to give univocal consequences in any possible case, not only as far as the derivation of p is concerned. So, we should look, not only for a univocal conclusion, but also for a univocal justification for it.
The notions just introduced seem intuitively satisfying, rigorously and declaratively defined, and able to account for the aspects of legal reasoning discussed in the examples above.
Let us consider the rules and exceptions of section 2.2. Let the legal qualification system Q include all those norms and the facts from **1** to **5**. Q is consistent, so that it has just one maximal consistent subset, to wit Q itself. The strong and weak legal consequences of Q coincide, being just the consequences of Q.
Let us add to Q the fact **6** so to obtain a premises set Q_1. Q_1 is inconsistent, but has just one preferred maximal subset, containing all facts and norms in Q_1 except ***fault_liability***. The statements asserting that Mark and John are liable are not derivable from that subset, so that they are not strongly nor weakly provable. But if we add to Q_2 the fact **7** we obtain a premises set Q_3, having again just one preferred subset, containing all statements in Q_3 except ***incapacity***. So we would derive the strong legal consequences that Mark is liable and that John is liable.
Let us add to our premises set, another principle, accepted by Italian judges, that is the principle that nobody can be considered liable for a fact that is not "attributable" to him, for which he is not somehow responsible. And the fact of the son is considered as not attributable to the parent, if the parent proves that he has given the son a good upbringing.

non_imputability(*X*, *Damage*, *Fact*):
¬ *liable_for*(*X*, *Damage*, *Fact*) ←
¬ *attributable*(*Fact*,*X*).

good_upbringing(*Par*, *Son*, *Fact*):
¬ *attributable*(*Fact*,*Par*) ←
parent(*Par*, *Son*) *accomplishes*(*Son*, *Fact*)
gave_a_good_upbringing(*Par*,*Son*).

If no priority relations among these norms and the parent liability rule are established (there is in fact such an uncertainty in Italian law), we have multiple preferred theories for the case that a well brought up son causes damage, one establishing that the parent is responsible, the other that he is not. In our example, given the fact
8:*gave_a_good_upbringing*(*mark*,*john*),

and assuming just the priority relations indicated in section 2.2, John's liability is a strong legal consequence, while Mark's liability is only a week legal consequence, *i.e.* a dubious one. The doubt can be eliminated by extending the < relation to establish, for example, as did most Italian judges, that ***parent_liability*** < ***non_imputability***.

Applicability

Another interesting notion that we can define in our approach is *applicability*. Intuitively, we can say that a norm is applicable in a case, if it can be used to derive a legal consequence in that case. This notion can be specified as follows (note the connection with the concepts of weak, strong, and strongly justified legal consequence, respectively):

- a norm n: $p \leftarrow q$ of a legal system A is *possibly applicable* in a certain case C, if there is a preferred subset B of A C (B $Pr(A$ $C)$), such that B contains n (n B) and B implies both q and p ($B \mid q, p$);
- n is *applicable* in C iff for all B $Pr(A$ $C)$, n B and $B \mid q, p$;
- n is *univocally applicable* if there is a B', such that B' B for all B $Pr(A$ $C)$, n B' and $B' \mid q, p$.

A difficult problem is connected with the fact that norms inconsistent with the preferred subsets including a certain case C cannot be applied in C and all their content (all their implications, also those consistent with the preferred subsets) are lost, as far as C (and cases containg C) is concerned.
In particular, rejecting all instances of a defeasible general statement, when just one of its instances is inconsistent with the preferred subset including C, would impede default reasoning, and lead to conclusions contrasting with our intuitions[30].
Brewka overcomes this last problem by representing general statements, not by universally quantified formulae, but by the corresponding open formulae, to be interpreted as the set of all their ground instances. So just the instances inconsistent with the preferred subsets are rejected. This solution can be generalized by putting A in clausal form, dropping universal quantifiers, and reading any open formula as the set of its ground instances[31].

7. Implicit abrogation

If normative conflicts can be solved in applying the law, then a legal system containing incompatible norms can be manageable. Therefore, there is no need to "postulate" the consistency of the legal system, that is to imagine a consistent system as the mysterious reality (to be recognized by the faculty of "legal interpretation") hiding behind the illusory phenomenon of inconsistency. Enactment and explicit abrogation of a norm n can be described simply as set addition and subtraction on the base A of the legal system (that is, as A $\{n\}$, and as A ¾ $\{n\}$, respectively)[32].
This model corresponds to the fact that the legislator normally adds (enacts) or retracts (abrogates) norms individually identified by their precise name (their positions in the text in which they are included). When he wants to eliminate a certain legal content p in circumstances q[33], he does it by enacting a norm n: $\neg p \leftarrow q$, stronger than any norm establishing p. In this way, he makes p no longer derivable in the case that q, but he also obtains an additional result: any future norms implying p, and weaker than n, will be blocked whenever q holds[34].
This approach to legal dynamics has a number of advantages but presents a serious handicap: the legal system is going to grow constantly, since no prescription is ever discarded from its base A, safe for the case of explicit abrogation, that is of a legislative act operating the subtraction of a determined statement from A.
In fact, jurists have found a remedy to this problem, by distinguishing two different

situations:

- *Partial incompatibility.* A norm n_1 is incompatible with stronger norms contained in the legal system, but there are possible cases (i.e. consistent sets of factual statements) in which which n_1 is applicable. The consequence of partial incompatibility is *derogation sensu stricto*: all incompatible norms are valid, and their conflicts are solved by means of the hierarchical ordering over the legal system. If a norm n_2 restricting the applicability of n_1 is removed from the system (or the applicability of n_2 is restricted), the applicability of n_1 expands correspondingly.
- *Total incompatibility*. After the enactment of a new norm n_2, there is no possible case in which a preexisting norm n_1 can be applied. The consequence of total incompatibility is *tacit abrogation*. The norm n_1 is definitively deleted from the system, and can no longer be recovered.

So, the distinction of derogation *sensu stricto* and tacit abrogation allows a compromise between two alternative strategies to face inconsistency: maintaining compatibility by eliminating the weakest incompatible norms, and accumulating all enacted norms in the system relying on their ordering to avoid inconsistent legal consequences.

8. Conclusion

We have seen that legal reasoning has often to deal with contradiction. The treatment of inconsistent information is not reducible to logical deduction, but, nevertheless, can partially be dealt with by means of formal methods. In contrast with the procedural models developed in other contributions (cf. for example Barklund and Hamfeld 1989) a fully declarative approach to the treatment of norm hierarchies has been proposed here.
We have considered only the dynamics of normative systems and the application of rules and exceptions, but other aspects of legal reasoning could be modelled as well by inconsistency handling, if the $\leq$ ordering over the legal norms is extended by further suitable criteria. Let us consider, for example, the choice among different interpretations. Let $p_1, \ldots, p_n$ be n different interpretations of the legal provision p (we can think of p as any statement included in a normative text), and let $p_1 <_E \ldots <_E p_n$ be their "hermeneutic" ordering ($<_E$ expresses the grading of their plausibility as interpretations of p, established by the interpreter according to his own hermeneutic criteria). Let us assume that the hierarchy $<A, \leq>$ is such that

a. A contains $p_1, \ldots, p_n$, and a further statement a_p, asserting that the $p_1, \ldots, p_n$ are alternative (just one of them can be chosen). The statement a_p can be thought of as the exclusive disjunction of the interpretations $p_1, \ldots, p_n$.
b. $\leq$ is extended accordingly to $p_1 <_E \ldots <_E p_n <_E a_p$ ($\leq_E$ is to be placed just after $\leq_{SR}$ in the priority grading of the criteria)[35].

Then, an interpretation p_i of a statement p in A can be said to be legally possible iff it is a weak legal consequence of A (*i.e.* if is included in a preferred subset), and legally preferable if it is a strong legal consequence of A (*i.e.* if is included in all preferred subsets)[36].
So, we obtain a partial formalization of the principle of systematic interpretation, prescribing to choose the interpretation that best fits in the system of legal norms. We also have some aspects of the principle of evolutionary interpretation: since the choice of p_i depends on the content of A, if A is modified (for example, if a higher level norm contradicting the previously chosen interpretation is enacted), then a different

interpretation can be selected.
In conclusion, the understanding of legal reasoning as contradiction handling seems able to model the actual reasoning of the jurist in a number of situations.
Possibly its relations with nonmonotonic reasoning, and in particular with abduction, will allow viable implementations in computer systems. But, for this purpose, further work is necessary.

9. Appendix: relations with other approaches

The treatment of legal hierarchies we have outlined in the previous pages can be easily related to the model of belief change proposed by Alchourrón, Gärdenfors, and Makinson, and to Poole's abduction[37].

1. THE AGM MODEL OF BELIEF CHANGE

Alchourrón, Gärdenfors, and Makinson (referred hereafter as AGM) have developed, a general theory of belief change, of "knowledge in flux"[38].
They consider three fundamental operations on a set of sentences:

- *Expansion:* a new proposition (axiom) *p,* hopefully consistent with a set *A*, is added to *A*, and this expanded set is closed under logical consequence.
- *Contraction:* a proposition *p*, derivable from *A*, is rejected from it, *i.e.* a set that does not imply *p* is obtained from *A*. This set, the contraction of *A* by *p*, is denoted *A* - *p*.
- *Revision:* a proposition *p,* inconsistent with *A*, is added to *A* with the requirement that the result be consistent and closed under logical consequence. The revision of *A* by *p* must contain *p* and must not imply ¬*p*.

Let us denote by $A \quad p$ the family of all maximal subsets $X \quad A$ such that $X \mid p$, *i.e.* such that *p* is not derivable from *X* under *Cn*.
Intuitively satisfying contraction functions can be characterized as *partial meet contractions, i.e.* by defining *A* - *p* as the intersection of a certain subset of $A \quad p$. This subset is specified by a selection function γ, which singles out the most important elements of $A \quad p$. The contraction function - is defined, so, as $\cap\gamma(A \quad p)$, *i.e. A* - *p* contains exactly the proposition common to all $X \quad \gamma(A \quad p)$.
The most interesting case is when γ is defined on the basis of a relation $\leq$ over 2^A, to wit when the selected elements of $A \quad p$ are defined to be the maximal under $\leq$:

$$\gamma(A \quad p) = \{B \quad (A \quad p) : B' \leq B \text{ , for all } B' \quad (A \quad p)\}.$$

The notions introduced in the previous sections can be related to the concept of contraction.
Let *A* be a set of statements. The family of the maximal (under) consistent subsets of *A* is $A \quad \boldsymbol{F}$ (where $\boldsymbol{F}$ denotes contradiction). Let us define the selection function γ on the basis of $\leq$, that is as

$$\gamma(A \quad p) = \{B \quad A \quad p : B' \leq B \text{ for all } B' \quad A \quad p\}.$$

So, $\gamma(A \perp \boldsymbol{F})$ contains the maximal (over $\perp$ and $\leq$) consistent subsets B of A, *i.e.* the preferred subtheories of A.
The strongly justified legal consequences of an inconsistent legal system A can be defined as the logical implications of the contraction of A by the false, that is as $Cn(A - \boldsymbol{F})$.
In fact, it easy to see that $A - \boldsymbol{F} \mid p$ iff a p is a strongly justified consequence of A, since $A - \boldsymbol{F}$ is defined as $\cap\gamma(A \perp \boldsymbol{F})$. Obviously, p is a strong consequence of A iff p is derivable from $\cap\{Cn(B): B \in \gamma(A \perp \boldsymbol{F})\}$.
If A is a theory, strong and strongly justified consequences coincide.

2. POOLE'S ABDUCTION

Let us recall that an abduction framework is defined (Poole 1988) by two sets of formulae:

a. A set $\hat{}$ of closed formulae called *facts*[39]. Facts are statements considered true, that we do do not intend to call into question.
b. A set Δ of formulae, called the set of *defaults*, or of possible *hypotheses*.[40]

Any ground instance of a formula in Δ can be used as a hypothesis to explain (justify) a goal (an abductive conclusion), if that instance is consistent with the facts. Let us indicate by G_Δ the set of the ground instances of the defaults in Δ. A goal is explainable from an abductive framework $< \hat{}, \Delta>$, iff it is implied by a consistent set (or *scenario*) $S = \hat{} \cup D$, where $D \subseteq G_\Delta$. The consequential closure of a maximal scenario is called an *extension.*
The concepts just introduced can easily be related to notions defined above.
Let the relation $<$ over $A = \hat{} \cup G_\Delta$ contain exactly the couples in $G_\Delta \times \hat{}$, and the relation $\leq$ over 2^A be defined on the basis of $<$.
Any scenario of the abduction framework $(\hat{}, \Delta)$ is a maximal, over $\leq$, consistent subset of A. So, any maximal, over $\perp$, scenario S of $< \hat{}, \Delta>$ is a preferred subset of A, *i.e.* $S \in P_R(A)$. Therefore, any statements p included in an extension of A is a weak consequence of A, *i.e.* there exists a $S \in P_R(A)$ such that $S \mid p$, while any statement q included in all extensions is a strong consequence of A, *i.e.* for all $S \in P_R(A), S \mid p$[41].

10. Notes

11. References

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1. For example, by the explicit abrogation of some regulations, or by the refutation of facts previously accepted as true.

2. By application area of a regulation (included in a normative system) we mean the set of cases in which the regulation is to be applied, to wit used in deriving legal consequences. The notion of applicability will be defined precisely in the following.

3. The result of the solution of the conflicts in a legal system A_1 can be expressed by building a consistent system A_2, such that the logical consequences of A_2 coincide with the legal consequences of A_1. But, as we shall see later, this coincidence cannot be exact, if the representation language has to be a "logic" in a strict sense. Moreover, the later changes of A_1 cannot be easily mapped into changes of A_2.

4. A case is a consistent set of factual statements, *i.e.* of statements in which predicates expressing legal qualifications do not occur (unless, perhaps, in modal contexts, such as the epistemic ones).

5. Let us define a logical consequence operation as "any operation *Cn* that takes sets of propositions to sets of propositions, such that three conditions are satisfied: $A \subseteq Cn(A)$ for any set A of propositions, known as *inclusion;* $Cn(A) = Cn(Cn(A))$, or *iteration;* and $Cn(A) \subseteq Cn(B)$ whenever $A \subseteq B$, or *monotony*" (Makinson 1985, 347). In the present paper we assume that *Cn*

includes first order classical logic, is compact ($p \in Cn(A')$ for some finite subset A' of A whenever $p \in Cn(A)$), and, finally, that it satisfies the rule of "introduction of disjunction in the premises", *i.e.* whenever $q \in Cn(A \cup \{p_1\})$ and $q \in Cn(A \cup \{p_2\})$, then $q \in Cn(A \cup \{p_1 \cup p_2\})$ (Makinson 1985, 348).

6. We assume that the variables in the norm are universally quantified (*i.e.* the norm is read as its universal closure). The representation of general defeasible norms will be considered in the following.

7. On naming, cf. Poole (1987;1988, 32ff).

8. Art. 1322 Italian Civil Code states that "the parties can freely determine the content of the contract ...". The Italian legal system (as any other legal system) limits this liberty by exceptions, as the same paragraph anticipates, continuing with "in the limits established by the law".

9. The provisions establishing criteria for the determination of the legal rent price are not relevant here.

10. We assume that legal norms can be ordered in a hierarchy (this concept will be specified in the following), and indicate by $n_1 < n_2$ the fact that, on the basis of that hierarchy, n_2 prevails over n_1.

11. The negation of the name of a rule is to be read as the statement of the inapplicability of that rule. So, this norm can be read as: "art 12 State Act 329/1978 is not applicable (to the rent price of a house) if the house is situated in a municipality having less than 20.000 inhabitants".

12. In Sartor (1991a,156-157) two kinds of exceptions are distinguished: *exceptions to norms* and *exceptions to effects.* Exceptions to norms state that particular norms, unambiguously identified, do not apply in a given situation. Exceptions to effects lay down that a particular legal qualification does not occur, is excluded, in a given situation. Exceptions to norms override the norm they refer to, exceptions to effects override any norm establishing the excluded effect. Here we represent exceptions to norms as conditional clauses whose conclusion is the negation of the name of the norm not to be applied, and exceptions to effects as clauses whose conclusion is the negation of the excluded effect.

13. The first two norms formalize a part of art. 2048 Italian Civil Code (*Parent liability* ...): "The father and the mother ... are liable for the damages caused by torts committed by their under age children ... [They] are freed from their liability only if they prove they could not prevent the fact."

14. This norm corresponds to art. 2043 Italian Civil Code (*Tort compensation*), establishing the general principle of fault liability: "Any intentional or negligent act, causing unjust damage to other persons, places any one who performs that act under a duty to pay compensation for damages".

15. This two last norms formalize art. 2046 Italian Civil code (*Liability for the harmful fact*): "He who was incapable at the time when he committed the harmful fact, is not liable for its consequences, unless the state of incapability derives

from his fault".

16. This reformulation corresponds to the (erroneous) opinion that legal norms are only *prima facie* incompatible: contradictions can be eliminated by means of interpretation, *i.e*. it is possible to obtain, via interpretation, a set of consistent norms such that all and only their logical consequences (under *Cn*) are the legal consequences of the contradictory norms. In fact, to obtain an equivalent representation without contradictions, we must go beyond logic (beyond *Cn* as defined above), and introduce a nonmonotonic formalism (cf. Sartor 1991a, 158 s).

17. For example, old norms establishing causes of justifications automatically limit the application of new crime provisions.

18. As in the example above, where the elimination of the constraint on rent prices determined the expansion of the principle of contractual freedom.

19. For example, Constitutional legislation prevails over ordinary Parliamentary legislation.

20. The notion of speciality is here understood in a wide sense, so as to include any type of relation between rule and exception.

21. In other words, the relations $\leq_{SR}$ and $\leq_T$ (defined as $n_1 \leq_{SR} n_2$ iff $n_1 <_{SR} n_2$ or $n_1 \ \mathsf{n}_{SR}\ n_2$ and $n_1 \leq_T n_2$ iff $n_1 <_T n_2$ or $n_1 \ \mathsf{n}_T\ n_2$) are connected.

22. More exactly, $\leq$ is a quasi-ordering, since it is not assumed to be antisymmetric.

23. It is sufficient to assume that no predicates expressing legal qualifications occur in the case, and that the norm is a hypothetical statement whose consequent expresses a consistent legal qualification. Under these assumptions there is always a consistent interpretation, to wit the interpretation making true all the facts of the case and also the legal qualification.

24. I recall that by *theory* here we mean any set if statements (not only deductive closures).

25. To be coherent with the treatment of hierarchies of legal rules developed in the following, the ordering of the levels in Brewka (1991, 72) has been inverted, and definitions have been changed correspondingly.

26. A similar idea is informally suggested by Sartor (1991a, 162-163).

27. Again, to be consistent with the following discussion, the < relation assumed in Brewka (1991, 75) has been inverted, and the definitions have been modified correspondingly.

28. As in Poole (1988, 29), general defeasible rules are represented as open formulae, to be interpreted as the set of their ground instances.

29. In fact, the families of preferred subsets obtained in the two approaches coincide.

30. Let us assume that in an assault both the assaulter and the assaulted person have been injured, the second having defended himself. The norm punishing personal injury cannot be applied to the assaulted person, since it is contradicted by the exception for self defence (blocking the instance of the rule concerning the assaulted), but clearly this contradiction should not impede the application of that criminal norm to the assaulter.

31. Possibly, more general results can be obtained by considering the preferred subsets, not of A, but of its deductive closure $Cn(A)$. But then other difficult problems have to be solved, such as the infinity of $Cn(A)$, and the extension of the $\leq$ relation to the statements not in A. On an ordering over theories, cf. Gärdenfors and Makinson (1988).

32. A can be considered as the set of the legal sources, or as their interpretation by the lawyer.

33. The condition q is empty (*i.e.* tautological) if the norm is categorical.

34. Constitutional norms establishing liberty rights are typical instances of this technique. In fact, we can consider strong permission Pq as equivalent to $\neg O \neg p$, *i.e.* as the negation of the obligation $O \neg p$.

35. The relation $<$ is defined so that $n_1 < n_2$ iff
    1. $n_1 <_{SR} n_2$; or
    2. $n_1 <_{E} n_2$, provided that $n_2 \ \mathsf{n}_{SR}\ n_1$; or
    3. $n_1 <_{S} n_2$, provided that $n_2 \ \mathsf{n}_{SR}\ n_1$ and $n_2 \ \mathsf{n}_{E}\ n_1$; or ...

36. It is easy to see that p_i, being in A is a strong legal consequence iff it is a strongly justified one.

37. The approach developed in the present paper, and in particular the relations with belief change, are expounded in Sartor (1991b).

38. Cf., for a systematic introduction, Makinson (1985), Gärdenfors (1988). The AGM model seems to have originated from the study of normative systems (cf. Alchourrón and Bulygin (1978, 1981), Alchourrón (1981), Alchourrón and Makinson (1982)).

39. We use underlining to distinguish Poole's concept of "fact" (a statement considered as certain) from the one used up to now, in the context of the norms-facts dichotomy.

40. Poole's model also comprises a third set of formulae, the constraints. In order to simplify the discussion, any reference to them will be omitted here.

41. The relation between abduction and theory revision is analysed in Gärdenfors and Makinson (1991).